Journal of Engineering Physics and Thermophysics, Vol. 77, No. 6, 2004

HEAT CONDUCTION AND HEAT EXCHANGE IN TECHNOLOGICAL PROCESSES

ENGINEERING METHODS OF CALCULATION OF DIFFERENT REGIMES OF HEATING OF THERMALLY MASSIVE OBJECTS IN METALLURGICAL HEAT TECHNOLOGIES UNDER COUNTERCURRENT CONDITIONS. 2. RADIATIVE AND RADIATIVE-CONVECTIVE COUNTERCURRENT HEATING

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UDC 669.046

An approximate procedure of mathematical modeling of radiative and radiative-convective countercurrent heat exchange in metallurgical units has been given.

Heat exchange of solid bodies and a gas in their countermotion is quite frequently realized in various branches of technology and primarily in metallurgy. Examples of such processes are heating of round billets in annular furnaces before piercing, heating of a burden in blast furnaces, cooling of pellets in stack-type coolers, etc. In exact solution, these problems are considered only in a linear formulation, whereas the processes themselves are nonlinear. This is because of the difficulties appearing in their nonlinear mathematical modeling. The arising complications can be successfully solved with the method of equivalent sources [1–3], which has been adequately tested in problems of concurrent and countercurrent heat exchange (CCHE).

Radiative Countercurrent Heat Exchange. Let us consider the problem of countercurrent symmetric radiative heating of thermally massive bodies of a base shape in the following formulation [1–3]:

$$\frac{1}{\rho^m}\frac{\partial}{\partial\rho}\left(\rho^m\frac{\partial\theta}{\partial\rho}\right)=\frac{\partial\theta}{\partial\tau}\,, \tag{1}$$

$$\left.\frac{\partial\theta}{\partial\rho}\right|_{\rho=1}=\text{Sk}\,[\theta_{\text{g}}^4(\tau)-\theta_{\text{s}}^4(\tau)]\,,\quad \left.\frac{\partial\theta}{\partial\rho}\right|_{\rho=0}=0\,, \tag{2}$$

$$\frac{d\theta_{\text{g}}}{d\tau}=\text{Sk}\,[\theta_{\text{g}}^4(\tau)-\theta_{\text{s}}^4(\tau)]\,n_m\,, \tag{3}$$

$$\theta(\rho,0)=\theta_0=\theta'=\text{const}\,,\quad \theta_{\text{g}}(0)=\theta''_{\text{g}}=1\,, \tag{4}$$

where we have

$$\theta_{\text{s}}(\tau)=\theta(1,\tau)\,;\quad \theta(\rho,\tau)=\frac{T_{\text{m}}(\rho,\tau)-T'_{\text{m}}}{T''_{\text{g}}-T'_{\text{m}}}\,;\quad n_m=(1+m)\,n\,;\quad \rho=\frac{r}{R}\,;$$

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$$\tau = \frac{at}{R^2}; \quad \text{Bi} = \frac{\alpha R}{\lambda}; \quad \theta(\rho, \tau) = \frac{T(\rho, \tau)}{T_g''}; \quad \theta_g(\tau) = \frac{T_g(\tau)}{T_g''}; \quad \text{Sk} = \frac{\sigma_v T_g'^3 R}{\lambda}. \tag{5}$$

The parameter n_m allows for the relation of the heat capacities of a solid body and a gas moving in opposition.

In the method of equivalent sources, which represents a combination of the method of successive approximations and the integral methods, it is taken that the inertial step of heating is completed after a certain time τ_0 and an ordered step of warmup over the entire body begins.

In the first inertial step of heating ($0 \le \tau \le \tau_0$ and $\beta(\tau) \le \rho \le 1$), we use the ready solution of problem (1), (2) by the method of equivalent sources [1–3]:

$$\theta_1(\rho, \tau) = \theta' + [\theta_{1s}(\tau) - \theta'] \left[\frac{\rho - \beta(\tau)}{1 - \beta(\tau)}\right]^2, \tag{6}$$

$$l(\tau) = 1 - \beta(\tau) = \frac{2}{\text{Sk}} \frac{\theta_{1s}(\tau) - \theta'}{\theta_{1g}^4(\tau) - \theta_{1s}^4(\tau)}. \tag{7}$$

The surface temperature $\theta_{1s}(\tau)$ or the temperature difference $\Delta\theta_1(\tau) = \theta_{1s}(\tau) - \theta'$ is determined by solution of the differential equation

$$\frac{d}{d\tau}[\Delta\theta_1(\tau)\, l(\tau)] = \frac{6(1 + m)\,\Delta\theta_1(\tau)}{l(\tau)}. \tag{8}$$

Equation (8) must be considered simultaneously with expression (7) and thermal-balance condition (3) but, taking into account the usual rapidity of the inertial period for the majority of metallurgical objects, we can simplify determination of the functions $\theta_{1s}(\tau)$ and $\theta_{1g}(\tau)$, assuming that the advance of the warmup front $l(\tau)$ satisfies a certain existing law, which is represented in this case by the formula [3]

$$l(\tau) = \sqrt{6(1 + n)\,\tau}, \quad \tau_0 = [6(1 + n)]^{-1}. \tag{9}$$

Therefore, the solution of Eq. (8) has the form

$$\Delta\theta_1(\tau) = \sqrt{\frac{\tau}{6(1 + m)}}, \quad \theta_{1s}(\tau) = \theta' + \sqrt{\frac{\tau}{6(1 + m)}}. \tag{10}$$

From relation (7) we find

$$\text{Sk}\,[\theta_{1g}^4(\tau) - \theta_{1s}^4(\tau)] = \frac{2\Delta\theta_1(\tau)}{l(\tau)}. \tag{11}$$

Then conditions (3) and (4) with account for (9)–(11) lead to the following expression for the gas temperature:

$$\theta_{1g}(\tau) = 1 + \frac{n}{3}\Delta\theta_1(\tau)\, l(\tau) = 1 + \frac{n\tau}{3}. \tag{12}$$

In the second (ordered) step ($\tau_0 \le \tau < \tau_*$ and $0 \le \rho \le 1$), the resolving equation of the method of equivalent sources is taken in the form of [1–3]. Integrating this equation [1–3] with respect to ρ and using boundary condition (2), we arrive at the solution

$$\theta_2(\rho, \tau) = \theta_{2s}(\tau) - \frac{\text{Sk}}{2}[\theta_{2g}^4(\tau) - \theta_{2s}^4(\tau)](1 - \rho^2) . \tag{13}$$

The relationship between the functions $f_2(\tau)$, $\theta_{2g}(\tau)$, and $\theta_{2s}(\tau)$ has the form $f_2(\tau) = -(1 + m)\ \text{Sk}\ [\theta_{2g}^4(\tau) - \theta_{2s}^4(\tau)]$.

Substituting expressions (12) and (13) into the integral condition [2, (23)], we have

$$(1 + m)\ \text{Sk}\ [\theta_{2g}^4(\tau) - \theta_{2s}^4(\tau)] = \frac{d}{d\tau}\left\{\theta_{2s}(\tau) - \frac{\text{Sk}}{3 + m}[\theta_{2g}^4(\tau) - \theta_{2s}^4(\tau)]\right\} \tag{14}$$

or, with the account for the heat-balance condition (13),

$$\frac{d\theta_{2g}(\tau)}{nd\tau} = \frac{d}{d\tau}\left\{\theta_{2s}(\tau) - \frac{\text{Sk}}{3 + m}[\theta_{2g}^4(\tau) - \theta_{2s}^4(\tau)]\right\} .$$

Integrating this equation, we obtain

$$\frac{1}{n}\theta_{2g}(\tau) + B = \theta_{2s}^4(\tau) - \frac{\text{Sk}}{3 + m}[\theta_{2g}^4(\tau) - \theta_{2s}^4(\tau)] . \tag{15}$$

The integration constant B is determined from the initial condition for the ordered step ($\tau = \tau_0$). The solution (13) for $\rho = 0$ yields

$$\text{Sk}\ [\theta_{2g}^4(\tau) - \theta_{2s}^4(\tau)] = 2\ [\theta_{2s}(\tau) - \theta_{2c}(\tau)] = 2\Delta\theta_2(\tau) . \tag{16}$$

Then expression (15) takes the form

$$\theta_{2g}(\tau) = \left[\theta_{2s}(\tau) - \frac{2}{3 + m}\Delta\theta_2(\tau) - B\right]n . \tag{17}$$

Equating the right-hand sides of formulas (12) and (17) at $\tau = \tau_0$, $l(\tau_0) = 1$, and $\theta_{2s}(\tau_0) = \theta_{2s}^0$, we find

$$B = \theta' - \frac{1}{n} + \frac{2m}{3\,(3 + m)}(\theta_{1s}^0 - \theta') . \tag{18}$$

From the solution (13), we determine the mass-mean temperature of the body:

$$\tilde{\theta}_2(\tau) = (1 + m)\int_0^1 \theta_2(\rho, \tau)\,\rho^m\, d\rho = \theta_{2s}(\tau) - \frac{\text{Sk}}{3 + m}[\theta_{2g}^4(\tau) - \theta_{2s}^4(\tau)] . \tag{19}$$

Comparing (15) and (19), we obtain

$$\theta_{2g}(\tau) - n\tilde{\theta}_2(\tau) = -Bn = \text{const} .$$

This result is consistent with the regularity noted in [4, 5]; it implies that in countercurrent, the equality

$$\theta_{2g}(\tau) = n\tilde{\theta}_2(\tau) + \theta_m , \tag{20}$$

where $\theta_m(T_m)$ is a certain arbitrary temperature (temperature parameter) having a constant value for the entire period of heating [6, p. 67], holds at any instant of heating time.

Let us determine from (18) the value of θ_m:

$$\theta_m = -Bn = 1 - n\theta' - \frac{2mn}{3\,(3 + m)}(\theta_{1s}^0 - \theta') , \tag{21}$$

which coincides with the expression $\theta_m = 1 - n\theta'$ given in [4] for $m = 0$ (for a plate). However, the value of the temperature parameter θ_m in [4] is true here only for a planar shape of a body. For other shapes we must use the general expression (21).

After eliminating the temperature function $\theta_{2s}(\tau)$ from (15), from condition (3) we find

$$\theta_{2s}(\tau) = \sqrt[4]{\theta_{2g}^4 - \frac{1}{n_m \mathrm{Sk}} \frac{\partial \theta_{2g}}{\partial \tau}} \,. \tag{22}$$

Then expression (15) with account for (21) and (22) takes the form

$$\theta_{2g}(\tau) - \theta_m = n\theta_{2g}(\tau) \sqrt[4]{1 - \frac{1}{n(1+m)\,\mathrm{Sk}\,\theta_{2g}^4(\tau)} \frac{\partial \theta_{2g}}{\partial \tau}} - \frac{1}{(1+m)(3+m)} \frac{\partial \theta_{2g}}{\partial \tau} \,. \tag{23}$$

The investigations have shown that the second term of the radicand for massive bodies (when $n\mathrm{Sk} > 0.1$) is less than unity. Taking this into account, we replace the radical involved in expression (23) by the first two terms of its power series. In this case we have the following differential equation:

$$\frac{1 + 4\,\mathrm{Sk}\,\theta_{2g}^3(\tau)/(3+m)}{[1 - k\theta_{2g}(\tau)]\,\theta_{2g}^3} d\theta_{2g} = 4(1+m)\,\theta_m\,\mathrm{Sk}\,d\tau \,, \tag{24}$$

where $k = (1 - n)/\theta_m$.

The integral of Eq. (24) will be represented by the transcendental relation

$$\Phi_g(\tau) - \Phi_g(\tau_0) = 4(1+m)\frac{\theta_m}{k^2}\mathrm{Sk}\,(\tau - \tau_0) \,, \tag{25}$$

in which

$$\Phi_g(\tau) = \ln \theta_{2g}(\tau) - p \ln [1 - k\theta_{2g}(\tau)] - \frac{0.5 + \theta_{2g}(\tau)}{k^2\theta_{2g}^2(\tau)} \,, \tag{26}$$

where $p = 1 + 4\ \mathrm{Sk}/[(3+m)k^3]$.

It is expressions (25) and (26) that determine the temperature function of the heat-transfer agent $\theta_{2g}(\tau)$ whose initial value for the ordered step [(12) at $\tau = \tau_0$] is already known.

To determine the temperature function of the surface $\theta_{2s}(\tau)$ from the gas temperature $\theta_{2g}(\tau)$ that is already known we use Eq. (15) with account for (18):

$$\theta_{2s}^4(\tau) + a_1\theta_{2s}(\tau) = a_0 \,, \tag{27}$$

where we introduce the notation

$$a_1 = \frac{3+m}{\mathrm{Sk}} \,, \quad a_0 = \theta_{2g}^4(\tau) + \frac{a_1}{n}[\theta_{2g}(\tau) - \theta_m] \,. \tag{28}$$

The algebraic equation (27) has a solution of the form

$$\theta_{2s}(\tau) = \frac{1}{2}\left(\sqrt{\frac{2a_1}{b_1} - b_1^2} - b_1\right), \quad b_1 = \sqrt{u + v} \,, \tag{29}$$

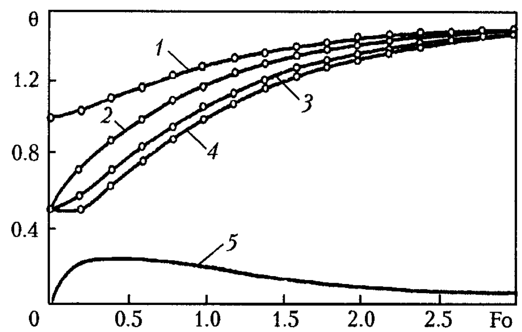

Fig. 1. Change in the temperature functions for the data of (35): 1) $\theta_g(\tau)$; 2) $\theta_s(\tau)$; 3) $\theta_c(\tau)$; 4) $\tilde{\theta}(\tau)$; 5) $\Delta\theta(\tau)$; points, results of [5].

where

$$\left.\begin{matrix} u \\ v \end{matrix}\right\} = \sqrt[3]{\frac{a_1^2}{2} \pm \sqrt{D}}\ ; \quad D = \left(\frac{4a_0}{3}\right)^3 + \left(\frac{a_1^2}{2}\right)^2 .$$

Knowing the functions of the temperatures $\theta_{2g}(\tau)$ and $\theta_{2s}(\tau)$, from the solution (13) for $\rho = 0$ we find the temperature function of the center $\theta_{2c}(\tau)$:

$$\theta_{2c}(\tau) = \theta_{2s}(\tau) - \frac{\text{Sk}}{2}\,[\theta_{2g}^4(\tau) - \theta_{2s}^4(\tau)] . \tag{30}$$

The mass-mean temperature of a body $\tilde{\theta}(\tau)$ is calculated from expression (19) or from (20) and (21):

$$\tilde{\theta}_2(\tau) = [\theta_{2g}(\tau) - \theta_m]/n . \tag{31}$$

Thus, the formulated problem (1)–(4) is solved.

The time τ_* of completion of heating is determined by solution of (25) under the assumption that

$$\theta_{2s}^*(\tau) = \theta_{2s}(\tau_*) = \eta\theta_{2g}(\tau_*) . \tag{32}$$

Substituting $\theta_{2s}^* = \eta\theta_{2g}^*$ into (27), we arrive at an algebraic equation of the same form:

$$\theta_{2g}^{*4} + a_{1g}\theta_{2g}^* = a_{0g}\,, \quad a_{1g} = \frac{3+m}{n\,\text{Sk}}\,\frac{1-n\eta}{1-\eta^4}\,, \quad a_{0g} = \frac{a_{1g} - \theta_m}{1-n\eta} . \tag{33}$$

Solution of Eq. (33) analogously to (29) determines the temperature θ_{2g}^* of the gas at the instant τ_* of completion of heating with a prescribed value η. Next, from expressions (25) and (26) we find

$$\tau_* = \tau_0 + \frac{k^2}{4\,(1+m)\,\theta_m\,\text{Sk}}\,(\Phi_g^* - \Phi_g(\tau_0)) . \tag{34}$$

Thus, we have obtained the analytical solution (generalized for all three shapes of base geometry) of the problem of countercurrent radiant heating of thermally massive bodies.

To evaluate the exactness of the solution obtained we have calculated the example taken from [5]:

$$m = 0\,, \quad n = 0.5\,, \quad \text{Sk} = 0.5\,, \quad \theta_0 = \theta' = 0.5\,, \quad \eta = 0.99 . \tag{35}$$

The results of the calculation are presented in Fig. 1.

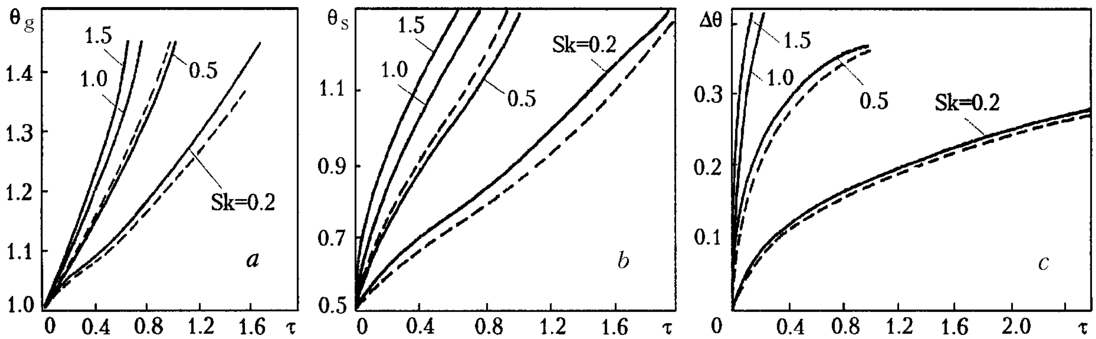

Fig. 2. Change in the temperature functions θ_g (a), θ_s (b), and $\Delta\theta$ (c) of a plate with Sk number for $n = 0.8$ and $\theta_0 = 0.5$; solid curves, method of equivalent sources, dashed curves, results of [5].

Based on the solution obtained in [6], we have investigated the dynamics of change of the functions $\theta_g(\tau)$, $\theta_s(\tau)$, and $\Delta\theta(\tau)$ in the process of heating of a plate as a function of the parameters Sk and n and for $\theta_0 = \theta'$. The results of the calculations are given in Fig. 2. The plots obtained by a numerical method in [5] are also given in Fig. 2 for the sake of comparison. Figure 3 shows the distribution function $F(\rho) = 1 - [\theta_s - \theta(\rho)]/(\theta_s - \theta_c)$ for two fixed instants of time referring to the inertial ($\tau = 0.1$) and ordered ($\tau = 0.4$) steps of heating.

A comparison to the data of [5] shows that the analytical solution proposed for the problem of radiative countercurrent heating of thermally massive bodies yields results with an exactness sufficient for practice.

In this solution, functions (6) and (13) explicitly express the coordinate dependence of the temperature field of a body, which enables one to apply this temperature function as a "load" function to investigation and calculation of the thermally stressed state of a body.

Radiative-Convective Countercurrent Heat Exchange. Countercurrent heating of a metal is carried out, as a rule, in an unheated (methodological) zone where heat exchange between the metal and low-temperature gases is carried out in addition to radiation and with a considerable fraction of the convective heat flux.

Let us consider the previous mathematical CCHE model in which the boundary conditions of radiative heating (2) and (3) will be replaced by the conditions of combined (radiative-convective) heat exchange

$$\left.\frac{d\theta}{d\rho}\right|_{\rho=1} = \mathrm{Sk}\left\{\theta_g^4(\tau) - \theta_s^4(\tau) + \zeta\,[\theta_g(\tau) - \theta_s(\tau)]\right\}, \quad (36)$$

$$\frac{d\theta_g}{d\tau} = \mathrm{Sk}\left\{\theta_g^4(\tau) - \theta_s^4(\tau) + \zeta\,[\theta_g(\tau) - \theta_s(\tau)]\right\} n_m\,, \quad (37)$$

where

$$\zeta = \mathrm{Bi}/\mathrm{Sk}\,; \quad \mathrm{Bi} = \frac{\alpha_{\mathrm{conv}} R}{\lambda}\,; \quad (38)$$

the remaining notation is the same as previously.

In the first (inertial) step ($0 \le \tau \le \tau_0$ and $\beta(\tau) \le \rho \le 1$), the solution by the method of equivalent sources has the same form (6), where

$$l(\tau) = \frac{2}{\mathrm{Sk}}\,\frac{\theta_{1s}(\tau) - \theta'}{\theta_{1g}^4(\tau) - \theta_{1s}^4(\tau) + \zeta\,[\theta_{1g}(\tau) - \theta_{1s}(\tau)]}\,. \quad (39)$$

The surface temperature (or the temperature difference) is also determined by the differential equation (8), which, after substitution of expression (39), takes the form

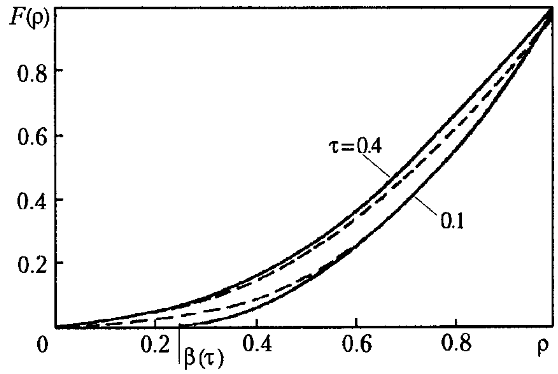

Fig. 3. Distribution of the temperature function $F(\rho, \tau)$ over the cross section of a plate for Sk = 0.5, n = 0.8, and θ_0 = 0.5; solid curves, method of equivalent sources, dashed curves, results of [5].

$$\frac{d}{d\tau}\frac{\Delta\theta_1^2(\tau)}{\theta_{1g}^4(\tau)-\theta_{1s}^4(\tau)+\zeta\,[\theta_{1g}(\tau)-\theta_{1s}(\tau)]}=\frac{3}{2}(1+m)\,\mathrm{Sk}^2\left\{\theta_{1g}^4(\tau)-\theta_{1s}^4(\tau)+\zeta\,[\theta_{1g}(\tau)-\theta_{1s}(\tau)]\right\}. \tag{40}$$

From the heat-balance condition (37), we find

$$\mathrm{Sk}\left\{\theta_{1g}^4(\tau)-\theta_{1s}^4(\tau)+\zeta\,[\theta_{1g}(\tau)-\theta_{1s}(\tau)]\right\}=\frac{1}{n_m}\frac{d\theta_{1g}}{d\tau}, \tag{41}$$

after which we can represent Eq. (40) as follows:

$$\frac{d}{d\tau}\frac{\Delta\theta_1^2(\tau)}{\theta_{1g}^4(\tau)-\theta_{1s}^4(\tau)+\zeta\,[\theta_{1g}(\tau)-\theta_{1s}(\tau)]}=\frac{3}{2}\frac{\mathrm{Sk}}{n}\frac{d\theta_{1g}}{d\tau}.$$

Integrating this expression and using initial conditions (4), we have

$$\frac{\Delta\theta_1^2(\tau)}{\theta_{1g}^4(\tau)-\theta_{1s}^4(\tau)+\zeta\,[\theta_{1g}(\tau)-\theta_{1s}(\tau)]}=\frac{3}{2}\frac{\mathrm{Sk}}{n}[\theta_{1g}(\tau)-1]. \tag{42}$$

From expression (39), it follows that

$$\theta_{1g}^4(\tau)-\theta_{1s}^4(\tau)+\zeta\,[\theta_{1g}(\tau)-\theta_{1s}(\tau)]=\frac{2\Delta\theta_1(\tau)}{\mathrm{Sk}\,l(\tau)},$$

after which we arrive at formula (12).

Let us take, as previously, that the advance of the warmup front $l(\tau)$ is described from (9), which results in formulas (10) and (12).

In the second (ordered) step ($\tau_0 \le \tau \le \tau_*$), according to the basic version of the method of equivalent sources, we obtain the solution [7, 8]

$$\theta_2(\rho,\tau)=\theta_{2s}(\tau)-\frac{\mathrm{Sk}}{2}\left\{\theta_{2g}^4(\tau)-\theta_{2s}^4(\tau)+\zeta\,[\theta_{2g}(\tau)-\theta_{2s}(\tau)]\right\}(1-\rho^2),$$
$$f_2(\tau)=-(1+m)\left\{\theta_{2g}^4(\tau)-\theta_{2s}^4(\tau)+\zeta\,[\theta_{2g}(\tau)-\theta_{2s}(\tau)]\right\}. \tag{43}$$

Substituting the functions of (43) into the condition [2, (23)], we arrive at the differential equation

$$(1+m)\,\mathrm{Sk}\left\{\theta_{2g}^{4}(\tau)-\theta_{2s}^{4}(\tau)+\zeta\,[\theta_{2g}(\tau)-\theta_{2s}(\tau)]\right\}=$$

$$=\frac{d}{d\tau}\left\{\theta_{2s}(\tau)-\frac{\mathrm{Sk}}{3+m}\,(\theta_{2g}^{4}(\tau)-\theta_{2s}^{4}(\tau)+\zeta\,[\theta_{2g}(\tau)-\theta_{2s}(\tau)])\right\}. \quad (44)$$

Taking into account the expression (41) resulting from condition (37), we bring Eq. (44) to the solution

$$\frac{\theta_{2g}(\tau)}{n}+B=\theta_{2s}(\tau)-\frac{\mathrm{Sk}}{1+m}\left\{\theta_{2g}^{4}(\tau)-\theta_{2s}^{4}(\tau)+\zeta\,[\theta_{2g}(\tau)-\theta_{2s}(\tau)]\right\}. \quad (45)$$

Setting $\rho = 0$ in $\theta_2(\rho, \tau)$ (expressions (43)), we find

$$\Delta\theta_2(\tau)=\theta_{2s}(\tau)-\theta_{2c}(\tau)=\frac{\mathrm{Sk}}{2}\left\{\theta_{2g}^{4}(\tau)-\theta_{2s}^{4}(\tau)+\zeta\,[\theta_{2g}(\tau)-\theta_{2s}(\tau)]\right\}.$$

Then, from relation (45), we have

$$\theta_{2g}(\tau)=\left[\theta_{2s}(\tau)-\frac{2}{3+m}\,\Delta\theta_2(\tau)-B\right]n\,. \quad (46)$$

Equating the right-hand sides of expressions (12) and (46) at $\tau = \tau_0$ and $l(\tau_0) = 1$, we arrive at the integration constant B determined by relation (18), after which we obtain

$$\theta_{2g}(\tau)=1-\frac{2mn}{3\,(3+m)}\,\Delta\theta_1^{(\tau_0)}+\frac{n}{3+m}\left\{(1+m)\,[\theta_{2s}(\tau)-\theta_{2c}(\tau)]+(3+m)\,[\theta_{2c}(\tau)-\theta']\right\}. \quad (47)$$

Let us find the mass-mean temperature of a body from the temperature function (43):

$$\tilde{\theta}_2(\tau)=(1+m)\int_0^1\theta_2(\rho,\tau)\,\rho^m d\rho=\theta_{2s}(\tau)-\frac{\mathrm{Sk}}{3+m}\left\{\theta_{2g}^{4}(\tau)-\theta_{2s}^{4}(\tau)+\zeta\,[\theta_{2g}(\tau)-\theta_{2s}(\tau)]\right\}. \quad (48)$$

Comparing expressions (45) and (48), we obtain the same formulas (20) and (21).

As we see, the law (20) is also true for a combined CCHT, and taking into account [9–13] devoted to a convective HT, it is true for any countercurrent heating irrespective of the form of a boundary condition. We only note that for other shapes of thermally massive bodies differing from a planar one the temperature parameter θ_m is determined by the more general expression (21).

Solving the problem further, from condition (37) we find the temperature function of the surface

$$\theta_{2s}(\tau)=\theta_{2g}(\tau)\,\sqrt[4]{1-\frac{1}{n_m\,\theta_{2g}^{4}(\tau)\,\mathrm{Sk}\left[1+\zeta\,\dfrac{\theta_{2g}(\tau)-\theta_{2s}(\tau)}{\theta_{2g}^{4}(\tau)-\theta_{2s}^{4}(\tau)}\right]}\,\frac{\partial\theta_{2g}(\tau)}{\partial\tau}}\,. \quad (49)$$

According to [3], we have

$$k_1=1+\zeta\,\frac{\theta_{2g}(\tau)-\theta_{2s}(\tau)}{\theta_{2g}^{4}(\tau)-\theta_{2s}^{4}(\tau)}\cong 1+\zeta\,\frac{0.275+0.058m}{\mathrm{Sk}}\,,$$

after which expression (49) takes the form

$$\theta_{2s}(\tau) = \theta_{2g}(\tau) \sqrt[4]{1 - \frac{1}{n_m k_1 \,\mathrm{Sk}\, \theta_{2g}^4(\tau)} \frac{\partial\theta_{2g}}{\partial\tau}} \; . \tag{50}$$

Thus, from equality (45) with account for (12), (21), and (50) we have

$$\theta_{2g}(\tau) - \theta_m = n\theta_{2g}(\tau) \sqrt[4]{1 - \frac{1}{n_m k_1 \,\mathrm{Sk}\, \theta_{2g}^4(\tau)} \frac{\partial\theta_{2g}(\tau)}{\partial\tau}} - \frac{1}{(1+m)(3+m)} \frac{\partial\theta_{2g}(\tau)}{\partial\tau} \, . \tag{51}$$

Replacing the radical involved in the right-hand side of expression (51) by the first two terms of its power series, we arrive at the differential equation with separable variables

$$\frac{1 + \dfrac{4k_1 \,\mathrm{Sk}\, \theta_{2g}^3(\tau)}{3+m}}{[1 - k\theta_{2g}(\tau)]\, \theta_{2g}^3(\tau)} d\theta_{2g} = 4\,(1+m)\, k_1 \theta_m \,\mathrm{Sk}\, d\tau \, . \tag{52}$$

After taking partial fractions of the rational-linear function on the left-hand side and integrating, we obtain a solution of the form (25):

$$\Phi_g(\tau) - \Phi_g(\tau_0) = 4\,(1+m) \frac{k_1 \theta_m \,\mathrm{Sk}}{k^2} (\tau - \tau_0) \, , \tag{53}$$

in which

$$\Phi_g(\tau) = \ln \theta_{2g}(\tau) - p \ln [1 - k\theta_{2g}(\tau)] - \frac{0.5 + \theta_{2g}(\tau)}{k^2 \theta_{2g}^2(\tau)} \, , \tag{54}$$

where $k = (1 - n)/\theta_m$ and $p = 1 + 4k_1 \,\mathrm{Sk}/[(3+m)k^3]$.

It is the transcendental equations (53) and (54) that determine the temperature function of the gas $\theta_{2g}(\tau)$ whose initial value $\theta_{1g}(\tau_0)$ is known [(12) at $\tau = \tau_0$].

Knowing the function $\theta_{2g}(\tau)$, we find the surface temperature $\theta_{2s}(\tau)$ of a body from expression (45), where

$$a_1 = \frac{3+m}{\mathrm{Sk}} + \zeta \, ; \quad a_0 = \theta_{2g}^4(\tau) + \frac{a_1}{n} \left[\left(1 + \zeta \frac{n\mathrm{Sk}}{3+m} \right) \theta_{2g}(\tau) - \theta_m \right] . \tag{55}$$

The temperature of the body's center is determined from expression (33) from the functions $\theta_{2g}(\tau)$ and $\theta_{2s}(\tau)$ (which are already known) for $\rho = 0$:

$$\Delta\theta_{2c}(\tau) = \theta_{2s}(\tau) - \frac{\mathrm{Sk}}{2} \left\{ \theta_{2g}^4(\tau) - \theta_{2s}^4(\tau) + \zeta\,[\theta_{2g}(\tau) - \theta_{2s}(\tau)] \right\} . \tag{56}$$

The mass-mean temperature of a body is calculated from expressions (48) or (20) and (21):

$$\tilde{\theta}_2(\tau) = \frac{\theta_{2g}(\tau) - \theta_c}{n} \, . \tag{57}$$

The time τ_* of completion of heating is found from the solution (43) and (44) under the assumption that $\theta_{2s}^* = \theta_{2s}(\tau_*) = \eta\theta_{2g}^*$.

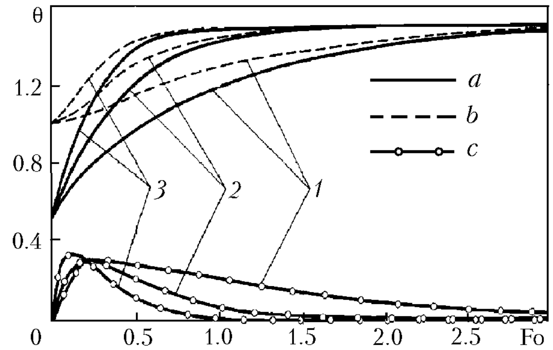

Fig. 4. Temperature dynamics of variously shaped bodies in radiant-convective CCHE (n = 0.5): 1) plate; 2) cylinder; 3) sphere; a) furnace (gas) temperature; b) surface temperature; c) temperature differences.

Substituting $\theta_{2s}^* = \eta\theta_{2g}^*$ into Eq. (27), we arrive at the same algebraic equation but now for θ_{2g}^*. New coefficients have the form

$$a_{1g} = \frac{3+m}{n\mathrm{Sk}}\frac{1-n\eta}{1-\eta^4}\left(1+\zeta\frac{n\mathrm{Sk}}{3+m}\frac{1-\eta}{1-n\eta}\right), \quad a_{0g} = \frac{(3+m)\,\theta_m}{n\mathrm{Sk}\,(1-\eta^4)} .$$

We compute the value of θ_{2g}^* from the solution analogous to (53) and (54), after which we find from Eq. (53) the required time

$$\tau_* = \tau_0 + \frac{k^2\,(\Phi_g'' - \Phi_g^0)}{4\,(1+m)\,k_1\,\mathrm{Sk}\,\theta_m} . \tag{58}$$

The formulated problem of radiative-convective heating of bodies under CCHE conditions is completely solved, which enables us to calculate the functions $\theta_g(\tau)$, $\theta_s(\tau)$, $\theta_c(\tau)$, $\bar{\theta}(\tau)$, $\Delta\theta(\tau)$, and the time τ_*.

To illustrate the use of the general solution obtained here we calculated [8] the heating of a plate for ζ = 1, Sk = 0.5, and n = 0.5 (Fig. 4).

The calculation results reflecting the dynamics of the process of heating of massive bodies of different geometries are presented in Fig. 4. The temperature difference over the cross section of an ingot is maximum at the end of the inertial step of heating.

An analysis of the results shows that in countercurrent, too, heating of a sphere (m = 2) is the most intense for the same characteristic dimension of the bodies. Decrease in the temperature difference in a plate is, conversely, the slowest process.

NOTATION

$a = \lambda/c\gamma$, thermal diffusivity, m^2/sec; Bi, Biot number; Fo, Fourier number; C, specific heat, J/(kg·K); D, integration constant; k, parameter; l, dimensionless thickness of the thermal layer; m, combining parameter of shape of a body (m = 0, plate; m = 1, cylinder, and m = 2, sphere); n, ratio of the water numbers of the material and the gas; n_m, refined value of the parameter n, allowing for the parameter of shape of a body; R, half the thickness of a plate, radius of a cylinder or a sphere, m; r, coordinate reckoned from the center of the body's cross section, m; Sk, Stark number; T, absolute temperature, K; t, time, sec; w, rate, m/sec; y, vertical coordinate, m; M, parameter; Φ, temperature function in the Kirchhoff substitution; α, heat-transfer coefficient, J/(m^3·K); β, coordinate of the warmed-up layer; γ, density, kg/m^3; $\eta = T_m''/T_g'$, degree of completeness of the heat-exchange process; λ, thermal conductivity, W/(m·K); μ, root of the equation; θ, relative excess temperature; $\bar{\theta}_m$, relative mass-mean temperature of a body; $\Delta\theta$, temperature difference; τ, dimensionless time; σ_v, visible coefficient of radiant heat exchange, W/(m^2·K^4); $\zeta = 1 - \rho$; ρ, dimensionless coordinate. Subscripts and superscripts: g, gas; m, material, solid body; s, surface of a body; m, ambient medium;

c, center of a body; *i*, No. of warmup step (inertial $i = 1$ or ordered $i = 2$); $'$ and $''$, values at entry and exit respectively; 0, initial value; *, completion of heating; conv, convective; v, visible.

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